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D-deformed Wess-Zumino model and its renormalizability properties

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Outline

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Deformed (noncommutative) space

Noncommutative space $\hat{\mathcal{A}}_{\hat{x}}$, generated by $\hat{x}^\mu$ coordinates $\mu = 0, 1, \dots n$ such that:

$$[\hat{x}^\mu, \hat{x}^\nu] = \Theta^{\mu\nu}(\hat{x}). \tag{1}$$

It is an associative free algebra generated by $\hat{x}^\mu$ and divided by the ideal generated by relations (1). [More formal: algebra of formal power series in h of polynomials in $\hat{x}$ or a completion of it.]
Three forms of $\Theta^{\mu\nu}(\hat{x})$ of special interest (Poincaré-Birkhoff-Witt property)

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}, \quad \theta^{\mu\nu} = -\theta^{\nu\mu} = const., \tag{2}$$

$$[\hat{x}^\mu, \hat{x}^\nu] = iC^{\mu\nu}_\lambda \hat{x}^\lambda, \quad C^{\mu\nu}_\lambda \text{ Lie algebra struct. const.,} \tag{3}$$

$$\hat{x}^\mu \hat{x}^\nu = \frac{1}{q} R^{\mu\nu}{}_{\rho\sigma} \hat{x}^\rho \hat{x}^\sigma, \quad R \text{ matrix of quantum group.} \tag{4}$$

$\star$-product approach

Idea: represent $\hat{\mathcal{A}}_{\hat{x}}$ on the space of commuting coordinates, but keep track of the deformation.

Poincaré-Birkhoff-Witt (PBW) theorem: ordering in the $\hat{\mathcal{A}}_{\hat{x}}$ has to be specified; symmetric ordering for example

$$
\begin{aligned}
:\hat{x}^\mu: &= \hat{x}^\mu, \\
:\hat{x}^\mu\hat{x}^\nu: &= \frac{1}{2}(\hat{x}^\mu\hat{x}^\nu + \hat{x}^\nu\hat{x}^\mu), \quad \text{etc.}
\end{aligned}
\qquad (5)
$$

PBW theorem:

$$
\begin{aligned}
:\hat{x}^\mu: &\mapsto x^\mu, \\
:\hat{x}^\mu\hat{x}^\nu: &\mapsto x^\mu x^\nu, \quad \text{etc.}
\end{aligned}
$$

$$
\begin{aligned}
\hat{f}(\hat{x}) &= C_0 + C_{1\mu} :\hat{x}^\mu: + C_{2\mu\nu} :\hat{x}^\mu\hat{x}^\nu: + \dots \\
&\downarrow \\
f(x) &= C_0 + C_{1\mu}x^\mu + C_{2\mu\nu}x^\mu x^\nu + \dots
\end{aligned}
\qquad (6)
$$

Also $$\hat{f}(\hat{x})\hat{g}(\hat{x}) = \hat{f}\cdot\hat{g}(\hat{x}) \mapsto f\star g(x). \qquad (7)$$

In the case of θ-deformed space

MW $\star$-product
$$f\star g\,(x) = \sum_{n=0}^{\infty}\Big(\frac{i}{2}\Big)^n \frac{1}{n!}\theta^{\rho_1\sigma_1}\dots\theta^{\rho_n\sigma_n}$$
$$\Big(\partial_{\rho_1}\dots\partial_{\rho_n}f(x)\Big)\Big(\partial_{\sigma_1}\dots\partial_{\sigma_n}g(x)\Big) \qquad (8)$$
$$= f\cdot g + \frac{i}{2}\theta^{\rho\sigma}(\partial_\rho f)\cdot(\partial_\sigma g) + \mathcal{O}(\theta^2).$$

Associative, noncommutative; c. conjugation: $(f\star g)^* = g^*\star f^*$.

Special example:
$$x^\mu\star x^\nu = x^\mu x^\nu + \frac{i}{2}\theta^{\mu\nu}$$
$$x^\nu\star x^\mu = x^\mu x^\nu + \frac{i}{2}\theta^{\nu\mu} = x^\mu x^\nu - \frac{i}{2}\theta^{\mu\nu}$$
$$[x^\mu \stackrel{\star}{,} x^\nu] = i\theta^{\mu\nu}. \qquad (9)$$

Twist formalism

- Motivation 1: Product (8) can be viewed as coming from an Abelian twist given by

$$\mathcal{F} = e^{-\frac{i}{2}\theta^{\rho\sigma}\partial_\rho\otimes\partial_\sigma} \tag{10}$$

as

$$\begin{aligned} f\star g &= \mu\{\mathcal{F}^{-1} f\otimes g\} \\ &= \mu\{e^{\frac{i}{2}\theta^{\rho\sigma}\partial_\rho\otimes\partial_\sigma} f\otimes g\} \\ &= f\cdot g + \frac{i}{2}\theta^{\rho\sigma}(\partial_\rho f)\cdot(\partial_\sigma g) + \mathcal{O}(\theta^2). \end{aligned} \tag{11}$$

- Motivation 2: Deformation $[x^\mu \stackrel{\star}{,} x^\nu] = i\theta^{\mu\nu}$ breaks the classical Lorentz symmetry.
  Is there a deformation of Lorentz symmetry such that it is a symmetry of (9)?

Basic idea and ingredients

Consider first a deformation (twist) of a classical symmetry (spacetime: Lorentz, SUSY or gauge). Then deform the spacetime itself.

Ingredients:

- Lie algebra g; generators t^a with $[t^a, t^b] = if^{abc}t^c$

  -Ξ: algebra of vector fields $u = u^\mu \partial_\mu$ with $[u, v] = uv - vu$.

- Universal enveloping algebra Ug

  -$U\Xi$: tensor algebra (over $\mathbb{C}$) generated by u and the unit element 1 modulo the left and right ideal generated by all elements $uv - vu - [u, v]$.

  -Ug is a Hopf algebra

$$\begin{aligned} \Delta(t^a) &= t^a \otimes 1 + 1 \otimes t^a \\ \varepsilon(t^a) &= 0, \quad S(t^a) = -t^a. \end{aligned} \tag{12}$$

Twist

A twist $\mathcal{F}$ (introduced by Drinfel'd in 1983-1985) is:

-an element of $Ug \otimes Ug$

-invertible

-fulfills the cocycle condition (ensures the associativity of the $\star$-product)

$$\mathcal{F} \otimes 1(\Delta \otimes id)\mathcal{F} = 1 \otimes \mathcal{F}(id \otimes \Delta)\mathcal{F}. \qquad (13)$$

-additionally: $\mathcal{F} = 1 \otimes 1 + \mathcal{O}(h)$; h-deformation parameter.

Notation: $\mathcal{F} = \mathrm{f}^{\alpha} \otimes \mathrm{f}_{\alpha}$ and $\mathcal{F}^{-1} = \bar{\mathrm{f}}^{\alpha} \otimes \bar{\mathrm{f}}_{\alpha}$.

An example of twist:

$$\mathcal{F} = e^{-\frac{i}{2}\theta^{ab}X_a \otimes X_b}, \qquad (14)$$

where $X_a = X_a^{\mu}(x)\partial_{\mu}$, $[X^a, X^b] = 0$ and $\theta^{ab} = const.$.

1. $\mathcal{F}$ applied to Ug: twisted Hopf algebra $Ug_{\mathcal{F}}$

$$
\begin{aligned}
[t^a, t^b] &= i f^{abc} t^c \\
\Delta_{\mathcal{F}}(t^a) &= \mathcal{F}\Delta(t^a)\mathcal{F}^{-1} \\
\varepsilon(t^a) &= 0, \quad S_{\mathcal{F}} = \mathrm{f}^\alpha S(\mathrm{f}_\alpha) S(t^a) S(\bar{\mathrm{f}}^\beta)\bar{\mathrm{f}}_\beta .
\end{aligned} \tag{15}
$$

2. $\mathcal{F}$ applied to $\mathcal{A}_x$ gives $\mathcal{A}_x^\star$

pointwise multiplication: $\mu(f \otimes g) = f \cdot g$

$\star$-multiplication:

$$
\begin{aligned}
\mu_\star(f \otimes g) &\equiv \mu \circ \mathcal{F}^{-1}(f \otimes g) \\
&= (\bar{\mathrm{f}}^\alpha f)(\bar{\mathrm{f}}_\alpha g) = f \star g .
\end{aligned} \tag{16}
$$

Comments I

Deformation by twist

Abelian twist $\mathcal{F} = e^{-\frac{i}{2}\theta^{\mu\nu}\partial_\mu\otimes\partial_\nu}, \quad \theta^{\mu\nu} = -\theta^{\nu\mu} \in \mathbb{R}$ leads to

1) θ-deformed Poincaré symmetry,

Chaichian et al. (Phys. Lett. B604, 98 (2004)), Wess (hep-th/0408080), Koch et al.(Nucl. Phys. B717, 387 (2005))

$$[\partial_\mu, \partial_\nu] = 0, \quad [\delta^\star_\omega, \partial_\rho] = \omega_\rho{}^\mu \partial_\mu, \quad [\delta^\star_\omega, \delta^\star_{\omega'}] = \delta^\star_{[\omega,\omega']},$$

$$\Delta(\delta^\star_\omega) = \delta^\star_\omega \otimes 1 + 1 \otimes \delta^\star_\omega + \frac{i}{2}\theta^{\rho\sigma}\Big(\omega^\lambda{}_\rho \partial_\lambda \otimes \partial_\sigma + \partial_\rho \otimes \omega^\lambda{}_\sigma \partial_\lambda\Big).$$

2) θ-deformed gravity,

Aschieri et al. (Class. Quant. Grav. 22, 3511 (2005) and 23, 1883 (2006))

$$[\delta^\star_\xi, \delta^\star_\eta] = \delta^\star_{[\xi,\eta]},$$

$$\Delta(\delta^\star_\xi) = \delta^\star_\xi \otimes 1 + 1 \otimes \delta^\star_\xi - \frac{i}{2}\theta^{\rho\sigma}\Big(\delta^\star_{(\partial_\rho\xi)} \otimes \partial_\sigma + \partial_\rho \otimes \delta^\star_{(\partial_\sigma\xi)}\Big) + \dots .$$

3) θ-deformed gauge theory,

Aschieri et al. (Lett. Math. Phys. 78 (2006) 61), Vassilevich (Mod. Phys. Lett. A 21 (2006) 1279), Giller et al. (Phys. Lett. B655, 80 (2007))

$$
\begin{aligned}
[\delta^\star_\alpha, \delta^\star_\beta] &= \delta^\star_{-i[\alpha,\beta]},\\
\Delta(\delta^\star_\alpha) &= \delta^\star_\alpha \otimes 1 + 1\otimes \delta^\star_\alpha - \frac{i}{2}\theta^{\rho\sigma}\Big(\delta^\star_{(\partial_\rho\alpha)}\otimes\partial_\sigma + \partial_\rho\otimes\delta^\star_{(\partial_\sigma\alpha)}\Big) + \dots .
\end{aligned}
$$

A more general Abelian twist $\mathcal{F} = e^{-\frac{i}{2}\theta^{ab}X_a\otimes X_b}$ leads to

4) nonconstant deformations, deformed gravity and SUGRA,

Aschieri et al. (Lett. Math. Phys. **85**, 39 (2008) and 0902.3817[hep-th],

0902.3823[hep-th]).

5) twisted supersymmetry,

P. Kosiński et al. (J. Phys. A **27** (1994) 6827), Y. Kobayashi et al. (Int. J. Mod. Phys. **A 20** (2005) 7175), Zupnik (Phys. Lett. **B 627** 208 (2005)), Ihl et al. (JHEP **0601** (2006) 065), . . .
M. Dimitrijević et al. (JHEP (2007))

$$
\begin{aligned}
\mathcal{F} &= e^{\frac{1}{2}C^{\alpha\beta}\partial_\alpha\otimes\partial_\beta + \frac{1}{2}\bar{C}_{\dot\alpha\dot\beta}\bar\partial^{\dot\alpha}\otimes\bar\partial^{\dot\beta}}, \quad C^{\alpha\beta} = C^{\beta\alpha}\in\mathbb{C}\\
[\delta^\star_\xi, \delta^\star_\eta] &= -2i(\eta\sigma^m\bar\xi - \xi\sigma^m\bar\eta)\partial_m,\\
\Delta(\delta^\star_\xi) &= \mathcal{F}\big(\delta^\star_\xi\otimes 1 + 1\otimes\delta^\star_\xi\big)\mathcal{F}^{-1}.
\end{aligned}
$$

D-deformed superspace

Undeformed superspace is generated by x^m, θ^α and $\bar\theta_{\dot\alpha}$ (anti)commuting coordinates

$$[x^m, x^n] = 0, \quad \{\theta^\alpha, \theta^\beta\} = 0, \quad \{\bar\theta_{\dot\alpha}, \bar\theta_{\dot\beta}\} = 0, \tag{17}$$

with $m = 0, \dots 3$ and $\alpha, \beta, \dot\alpha, \dot\beta = 1, 2$.

Superfiled $F(x, \theta, \bar\theta)$ can be expanded in powers of θ and $\bar\theta$

$$\begin{aligned} F(x,\theta,\bar\theta) \;=\; & f(x) + \theta\phi(x) + \bar\theta\bar\chi(x) + \theta\theta m(x) + \bar\theta\bar\theta n(x) \\ & + \theta\sigma^m\bar\theta v_m + \theta\theta\bar\theta\bar\lambda(x) + \bar\theta\bar\theta\theta\varphi(x) + \theta\theta\bar\theta\bar\theta d(x). \end{aligned} \tag{18}$$

Under infinitesimal SUSY transformations

$$\delta_\xi F = \big(\xi Q + \bar\xi\bar Q\big) F, \qquad (19)$$

$$Q_\alpha = \partial_\alpha - i\sigma^m{}_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\,\partial_m\,, \quad \bar Q^{\dot\alpha} = \bar\partial^{\dot\alpha} - i\theta^\alpha\sigma^m{}_{\alpha\dot\beta}\varepsilon^{\dot\beta\dot\alpha}\partial_m. \qquad (20)$$

Also $\xi^a, \bar\xi_{\dot\alpha} = const.$ and $\{\xi^\alpha, \xi^\beta\} = \{\xi^\alpha, \bar\xi_{\dot\alpha}\} = \{\bar\xi_{\dot\alpha}, \bar\xi_{\dot\beta}\} = 0$.

Transformations (5) close in the algebra

$$[\delta_\xi, \delta_\eta] = -2i\big(\eta\sigma^m\bar\xi - \xi\sigma^m\bar\eta\big)\partial_m. \qquad (21)$$

Leibniz rule is undeformed

$$\begin{aligned}\delta_\xi\big(F\cdot G\big) &= (\delta_\xi F)\cdot G + F\cdot(\delta_\xi G)\\ &= \big(\xi Q + \bar\xi\bar Q\big)\big(F\cdot G\big).\end{aligned} \qquad (22)$$

Deformation: we use the twist approach and choose the twist

$$\mathcal{F} = e^{\frac{1}{2}C^{\alpha\beta}D_\alpha\otimes D_\beta}, \tag{23}$$

with $C^{\alpha\beta} = C^{\beta\alpha} \in \mathbb{C}$ and $D_\alpha = \partial_\alpha - i\sigma^{\alpha\dot\alpha}_m\bar\theta_{\dot\alpha}\partial_m$.
Then $\mathcal{F}^{-1} = e^{-\frac{1}{2}C^{\alpha\beta}D_\alpha\otimes D_\beta}$ defines the $\star$-product

$$\begin{aligned} F\star G &= \mu\{\mathcal{F}^{-1}\, F\otimes G\} \\ &= F\cdot G - \frac{1}{2}(-1)^{|F|}C^{\alpha\beta}(D_\alpha F)\cdot(D_\beta G) \\ &\quad -\frac{1}{8}C^{\alpha\beta}C^{\gamma\delta}(D_\alpha D_\gamma F)\cdot(D_\beta D_\delta G), \end{aligned} \tag{24}$$

where $|F| = 1$ if F is odd (fermionic) and $|F| = 0$ if F is even (bosonic). The $\star$-product is associative, non(anti)commutative and non-hermitian, $(F\star G)^* \neq G^*\star F^*$.

Especially,

$$\{\theta^\alpha \stackrel{\star}{,} \theta^\beta\} = C^{\alpha\beta}, \quad \{\bar\theta_{\dot\alpha} \stackrel{\star}{,} \bar\theta_{\dot\beta}\} = \{\theta^\alpha \stackrel{\star}{,} \bar\theta_{\dot\alpha}\} = 0,$$
$$[x^m \stackrel{\star}{,} x^n] = -C^{\alpha\beta}(\sigma^{mn}\varepsilon)_{\alpha\beta}\bar\theta\bar\theta, \dots . \tag{25}$$

Non-anticommutative superspace: non-anticommutativity is encoded in terms of the $\star$-product (24).
Since $\{Q_\alpha, D_\beta\} = \{\bar Q_{\dot\alpha}, D_\beta\} = 0$ the Hopf algebra of SUSY transformations does not change, it remains undeformed:

$$\delta^\star_\xi F = \big(\xi Q + \bar\xi\bar Q\big) F,$$
$$\delta^\star_\xi (F \star G) = \big(\xi Q + \bar\xi\bar Q\big)(F \star G)$$
$$= (\delta^\star_\xi F) \star G + F \star (\delta^\star_\xi G). \tag{26}$$

Deformed Wess-Zumino model

Chiral field Φ fulfills $\bar{D}_{\dot\alpha}\Phi = 0$, with $\bar{D}_{\dot\alpha} = -\bar\partial_{\dot\alpha} - i\theta^\alpha\sigma^m_{\alpha\dot\alpha}\partial_m$

$$\Phi(x) = A(x) + \sqrt{2}\theta^\alpha\psi_\alpha(x) + \theta\theta\mathsf{F}(x) + i\theta\sigma^l\bar\theta(\partial_l A(x)) - \frac{i}{\sqrt{2}}\theta\theta(\partial_m\psi^\alpha(x))\sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha} + \frac{1}{4}\theta\theta\bar\theta\bar\theta(\Box A(x)). \tag{27}$$

The $\star$-product of two, three,… chiral superfields is not chiral and we use the projectors

$$P_1 = \frac{1}{16}\frac{D^2\bar{D}^2}{\Box}, \quad P_2 = \frac{1}{16}\frac{\bar{D}^2D^2}{\Box}, \quad P_T = -\frac{1}{8}\frac{D\bar{D}^2D}{\Box}, \tag{28}$$

$$f(x)\frac{1}{\Box}g(x) = f(x)\int \mathrm{d}^4y\; G(x-y)g(y)$$

to separate chiral and antichiral parts of $\star$-products of Φ. For example

$$P_2(\Phi\star\Phi) = \Phi\Phi = A^2 + 2\sqrt{2}A\theta^\alpha\psi_\alpha + \theta\theta\Big(2AH - \psi\psi\Big) + i\theta\sigma^m\bar\theta\Big(\partial_m(A^2)\Big) + i\sqrt{2}\theta\theta\bar\theta_{\dot\alpha}\bar\sigma^{m\dot\alpha\alpha}\big(\partial_m(\psi_\alpha A)\big) + \frac{1}{4}\theta\theta\bar\theta\bar\theta\Box A^2. \tag{29}$$

Undeformed Wess-Zumino Lagrangian

$$\mathcal{L} = \Phi^+ \cdot \Phi\Big|_{\theta\theta\bar{\theta}\bar{\theta}} + \Big(\frac{m}{2}\Phi\cdot\Phi\Big|_{\theta\theta} + \frac{\lambda}{3}\Phi\cdot\Phi\cdot\Phi\Big|_{\theta\theta} + \text{ c.c. }\Big), \qquad (30)$$

with m and λ real constants.

Deformation 1:

$$\begin{aligned}
\Phi^+\cdot\Phi &\to \Phi^+\star\Phi,\\
\Phi\cdot\Phi &\to P_2\big(\Phi\star\Phi\big),\\
\Phi\cdot\Phi\cdot\Phi &\to \begin{cases} P_2\Big(\Phi\star P_2\big(\Phi\star\Phi\big)\Big),\\ P_2\Big(P_2\big(\Phi\star\Phi\big)\star\Phi\Big),\\ P_2\Big(\Phi\star\Phi\star\Phi\Big).\end{cases}
\end{aligned}$$

Deformation 2: Use all possible SUSY covariant terms in two and three fields.

The second approach gives

$$\mathcal{L} = \Phi^+ \star \Phi\Big|_{\theta\theta\bar\theta\bar\theta} + \Big\{ \frac{m}{2}\Big(P_2(\Phi\star\Phi)\Big|_{\theta\theta} + 2a_1 P_1(\Phi\star\Phi)\Big|_{\bar\theta\bar\theta}\Big)$$
$$+\frac{\lambda}{3}\Big(P_2(P_2(\Phi\star\Phi)\star\Phi)\Big|_{\theta\theta} + 3a_2 P_1(P_2(\Phi\star\Phi)\star\Phi)\Big|_{\bar\theta\bar\theta}$$
$$+2a_3(P_1+P_2)(P_1(\Phi\star\Phi)\star\Phi)\Big|_{\theta\theta\bar\theta\bar\theta}\Big) + \text{ c.c. } \Big\}. \quad (31)$$

The D-deformed Wess-Zumino action in component fields

$$S = \int \mathrm{d}^4x\, \Big\{ A^*\Box A + i\partial_m\bar\psi\bar\sigma^m\psi + H^*H + m(AH - \frac{1}{2}\psi\psi) + m(A^*H^* - \frac{1}{2}\bar\psi\bar\psi)$$
$$+\lambda(A^2H - A\psi\psi) + \lambda\big((A^*)^2H^* - A^*\bar\psi\bar\psi\big)$$
$$+C^2\Big(ma_1(\frac{1}{2}\psi\Box\psi - H\Box A) + \lambda a_2(-AH\Box A - \frac{1}{2}H(\Box A^2)$$
$$+\frac{1}{2}\psi\psi(\Box A) + A\psi\Box\psi) + \lambda a_3(-\frac{3}{2}H^2\Box A + \frac{3}{2}H(\partial_m\psi)\sigma^m\bar\sigma^l(\partial_l\psi)\Big)$$
$$+\bar C^2\Big(\text{ c.c. }\Big)\Big\}. \quad (32)$$

From the action (32) one can:

-calculate the EOM for the fields H and H^*, solve them and insert the solutions back into the action.

-look at the renormalizability properties (motivation: twisted symmetry vs. renormalization).

Renormalizability properties: use the background field method to calculate the divergences in the two-point functions.

- Revrite the action (32) in terms of the real fields.
- Split the fields into their classical and quantum parts.
- Calculate the action quadratic in quantum fields

$$S^{(2)} = \frac{1}{2} \begin{pmatrix} \bar{\Psi} \, \mathcal{S} \, \mathcal{P} \, \mathcal{E} \, \mathcal{G} \end{pmatrix} M \begin{pmatrix} \Psi \\ \mathcal{S} \\ \mathcal{P} \\ \mathcal{E} \\ \mathcal{G} \end{pmatrix} . \qquad (33)$$

- The one loop effective action is

$$\Gamma = \frac{i}{2}\mathrm{STr}\ln\Big[1+(\Box-m^2)^{-1}MC\Big], \qquad (34)$$

with

$$MC = N+T+V. \qquad (35)$$

- Calculate (34) up to second order in g, second order in fields and up to second order in $C_{\alpha\beta}$

$$\begin{aligned}\Gamma &= \frac{i}{2}\mathrm{STr}\ln\Big[1+(\Box-m^2)^{-1}(N+T+V)\Big]\\ &= \frac{i}{2}\Big[\mathrm{STr}((\Box-m^2)^{-1}(N+T+V))\\ &\quad -\frac{1}{2}\mathrm{STr}((\Box-m^2)^{-1}N(\Box-m^2)^{-1}N)\\ &\quad -\mathrm{STr}((\Box-m^2)^{-1}N(\Box-m^2)^{-1}(T+V))\\ &\quad +\mathrm{STr}(((\Box-m^2)^{-1}N)^2(\Box-m^2)^{-1}T)\Big]. \qquad (36)\end{aligned}$$

- Calculate Supertraces…

Results

- The divergent part of the one loop effective action is

$$\begin{aligned}\Gamma_1 \; = \; & \frac{g^2}{\pi^2\epsilon}\int \mathrm{d}^4x\,\Big[\frac{1}{4}(S\Box S + P\Box P + \bar{\psi}i\partial\!\!\!/\psi + E^2 + G^2) \\ & + \frac{3}{4}a_3C^2m^2(2P\Box G + \bar{\psi}\Box\psi - 2S\Box E) \\ & - C^2a_1m^2(S\Box S + P\Box P - \bar{\psi}i\partial\!\!\!/\psi + E^2 + G^2)\Big]. \end{aligned} \quad (37)$$

- To cancel the divergences we have to add counterterms to the classical Lagrangian

$$\mathcal{L}_B = \mathcal{L}_0 + \mathcal{L}_2 - \Gamma_1. \quad (38)$$

- All the fields are renormalized in the same way:

$$S_0 = \sqrt{Z}S,\ P_0 = \sqrt{Z}P,\ \psi_0 = \sqrt{Z}\psi,\ E_0 = \sqrt{Z}E,\ G_0 = \sqrt{Z}G,$$
$$Z = 1 - \frac{g^2}{2\pi^2\epsilon}(1 - 4a_1 m^2 C^2). \tag{39}$$

- The tadpole contributions add up to zero; $\delta m = 0$; the deformation parameter has to be renormalized

$$C_0^2 = \left(a_1 - \frac{3a_3 g^2}{2\pi^2\epsilon}\right)C^2\ . \tag{40}$$

- No deformation of the nonrenormalization theorem.

Comments II

- A deformation of the superspace is constructed:
  -the $\star$-product is not hermitian
  -the SUSY Hopf algebra is undeformed, especially the Leibniz rule does not change.
- Deformed Wess-Zumino model is discussed:
  -all possible covariant terms are included in the action
  -divergences of two-point Green functions: no mass renormalization, all fields are renormalized in the same way
  -no additional terms needed in the original action in order to absorb the divergences.
- future work/work in progress:
  -renormalizability of the model continued: three point functions
  -vector superfield, that is gauge theories.