STATUS OF THE MINIMAL SUPERSYMMETRIC *SO*(10) MODEL*

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*Borut Bajc, I.D. i Miha Nemevšek, arXiv:0809.1069, **JHEP** 0811:007, 2008.

OUTLINE

- **MINIMAL** *SO*(10) **MODEL: MAIN FEATURES**
- **LOW SUPERSYMMETRIC SCALE & NEUTRINO MASS**
- **SPLIT SUPERSYMMETRY & NEUTRINO MASS**
- **EXPERIMENTAL SIGNALS**
- **CONCLUSIONS**

MINIMAL $SO(10)$ — MSG(rand)U(nified)T(heory)*

REPRESENTATIONS: $16_{F\,i}, (i = 1, 2, 3)$ 210_H 126_H $\overline{126}_H$ 10_H

SUPERPOTENTIAL: $W_Y = 16_F(Y_{10}10_H + Y_{\overline{126}}\overline{126}_H)16_F$

(W_Y is relevant for the description of fermion masses!
Y_{10} and $Y_{\overline{126}}$ are 3 × 3 symmetric Yukawa matrices.)

$$W_H = \frac{m}{4!}210_H^2 + \frac{\lambda}{4!}210_H^3 + \frac{M}{5!}126_H\overline{126}_H + \frac{\eta}{5!}126_H 210_H \overline{126}_H + m_H {10_H}^2 + \frac{1}{4!}210_H 10_H(\alpha 126_H + \overline{\alpha}\overline{126}_H)$$

(W_H is relevant for the gauge coupling unification (α_i, $i = 1, 2, 3$)!)

*T. E. Clark, T. K. Kuo and N. Nakagawa, Phys. Lett. B 115 (1982) 26; K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. 70 (1993) 2845; C. S. Aulakh, B. Bajc, A. Melfo, G. Senjanović and F. Vissani, Phys. Lett. B 588 (2004) 196; …

RELEVANT PARAMETERS: (W_Y)

EXPERIMENTALLY DETERMINED QUANTITIES		MODEL
3 masses of "up" quarks:	m_u, m_c, m_t	Y_{10} $Y_{\overline{126}}$
3 masses of "down" quarks:	m_d, m_s, m_b	
3 masses of charged leptons:	m_e, m_μ, m_τ	
2 parameters (3 masses) of neutral leptons:	m_1, m_2, m_3	
3 CKM* angles + 1 CKM phase:	$\theta^{CKM}_{12,\,23,\,13}, \delta^{CKM}$	3 + 12
2 (3) PMNS† angles + 0 (3) PMNS phases:	$\theta^{PMNS}_{12,\,23,\,13}, \ldots$	
17 (22)		15

*Cabibbo-Kobayashi+-Maskawa+ (CKM) (+Nobel Prize in Physics 2008)

†Pontecorvo-Maki-Nakagawa-Sakata (PMNS)

RELEVANT PARAMETERS: (W_H)

CONDITIONS*: M, m_H, m ➡ $M = m\frac{\eta}{\lambda}\frac{3-14x+15x^2-8x^3}{(x-1)^2}$ ➡ m

⬇

PARAMETERS*:

$m, \quad \alpha, \quad \overline{\alpha}, \quad |\lambda|, \quad |\eta|, \quad \phi = \arg(\lambda) = -\arg(\eta), \quad x = \mathrm{Re}(x) + i\mathrm{Im}(x), \quad g_{GUT}, \quad M_{SUSY}$

($\phi \to 0$)

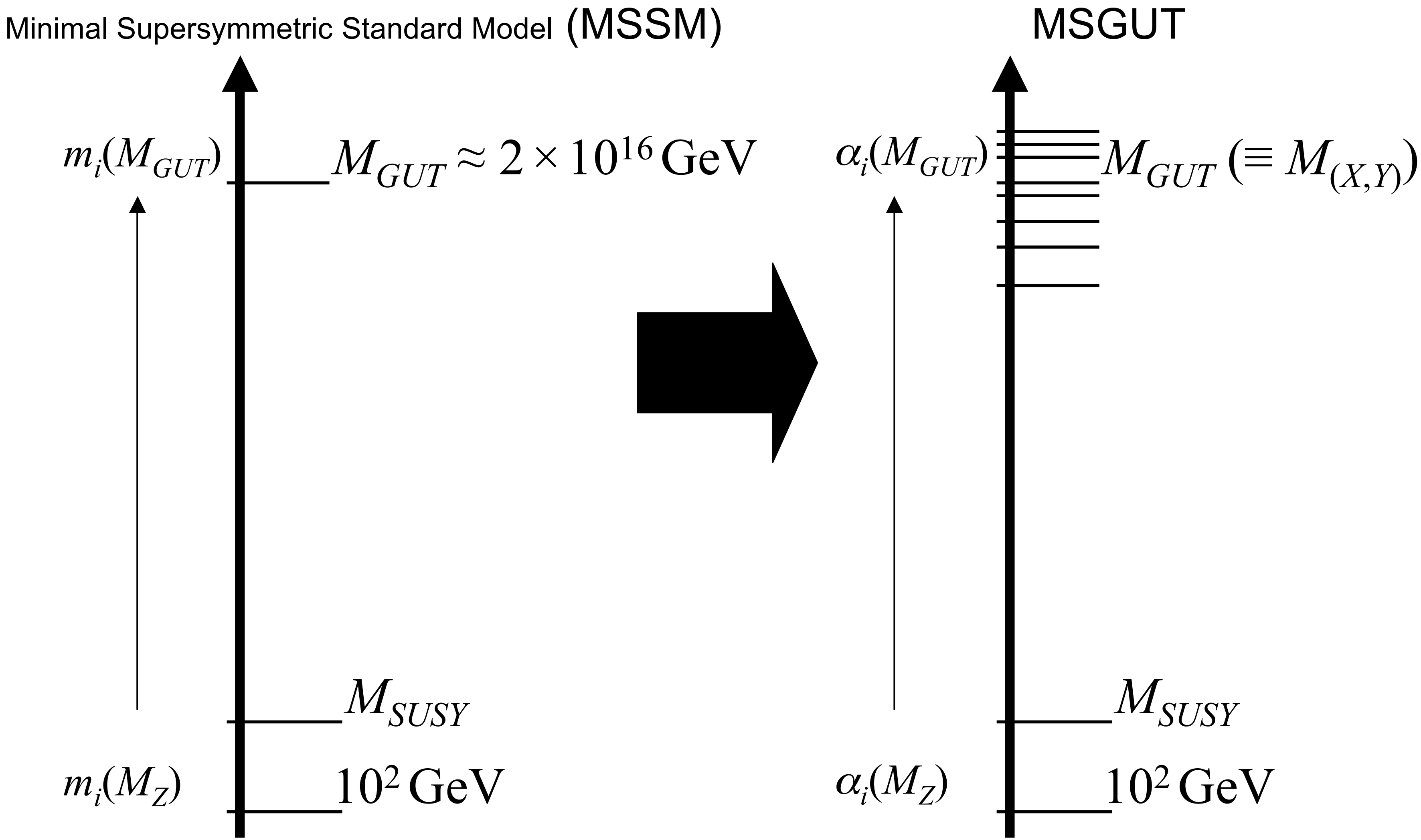
These parameters are important for the running of gauge couplings.

$\tan\beta$

These parameters are important for the running of Yukawa couplings.

*C. S. Aulakh, B. Bajc, A. Melfo, G. Senjanović and F. Vissani, Phys. Lett. B 588 (2004) 196.

UNIFICATION

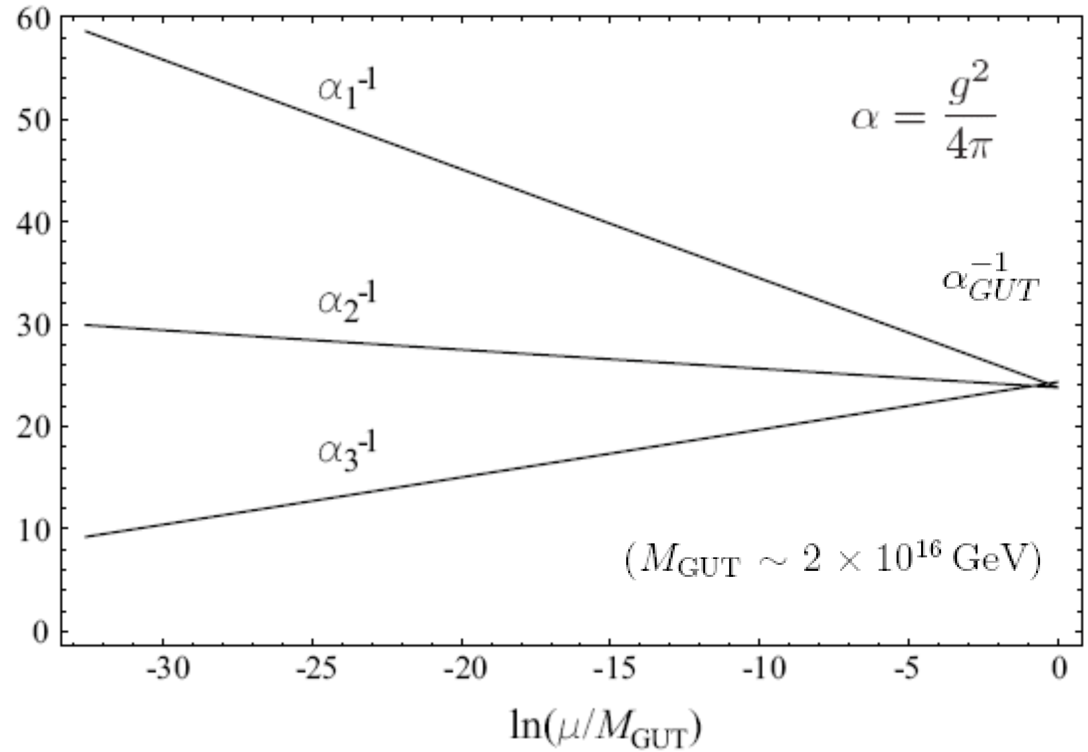


MSSM: GAUGE COUPLING UNIFICATION

$\alpha_1(M_Z) = 0.016949 \pm 0.000005$

$\alpha_2(M_Z) = 0.033816 \pm 0.000027$

$\alpha_3(M_Z) = 0.1176 \pm 0.0020$



MSGUT: GAUGE COUPLING UNIFICATION*

?

*B. Bajc et al, hep-ph/0402122; C. S. Aulakh and S. K. Garg, hep-ph/0512224; S. Bertolini, T. Schwetz and M. Malinsky, hep-ph/0605006; …

FERMION MASSES IN MINIMAL *SO*(10)*

$$M_u = \frac{N_u}{N_d}\tan\beta[M_d + \xi(M_d - M_e)]$$

$$M_n = \frac{v}{m}\frac{\sin^2\beta}{\cos\beta}\alpha\sqrt{\frac{|\lambda|}{|\eta|}}\frac{N_u^2}{N_d}\left[m_I\, f_I + m_{II}\, f_{II}\right]$$

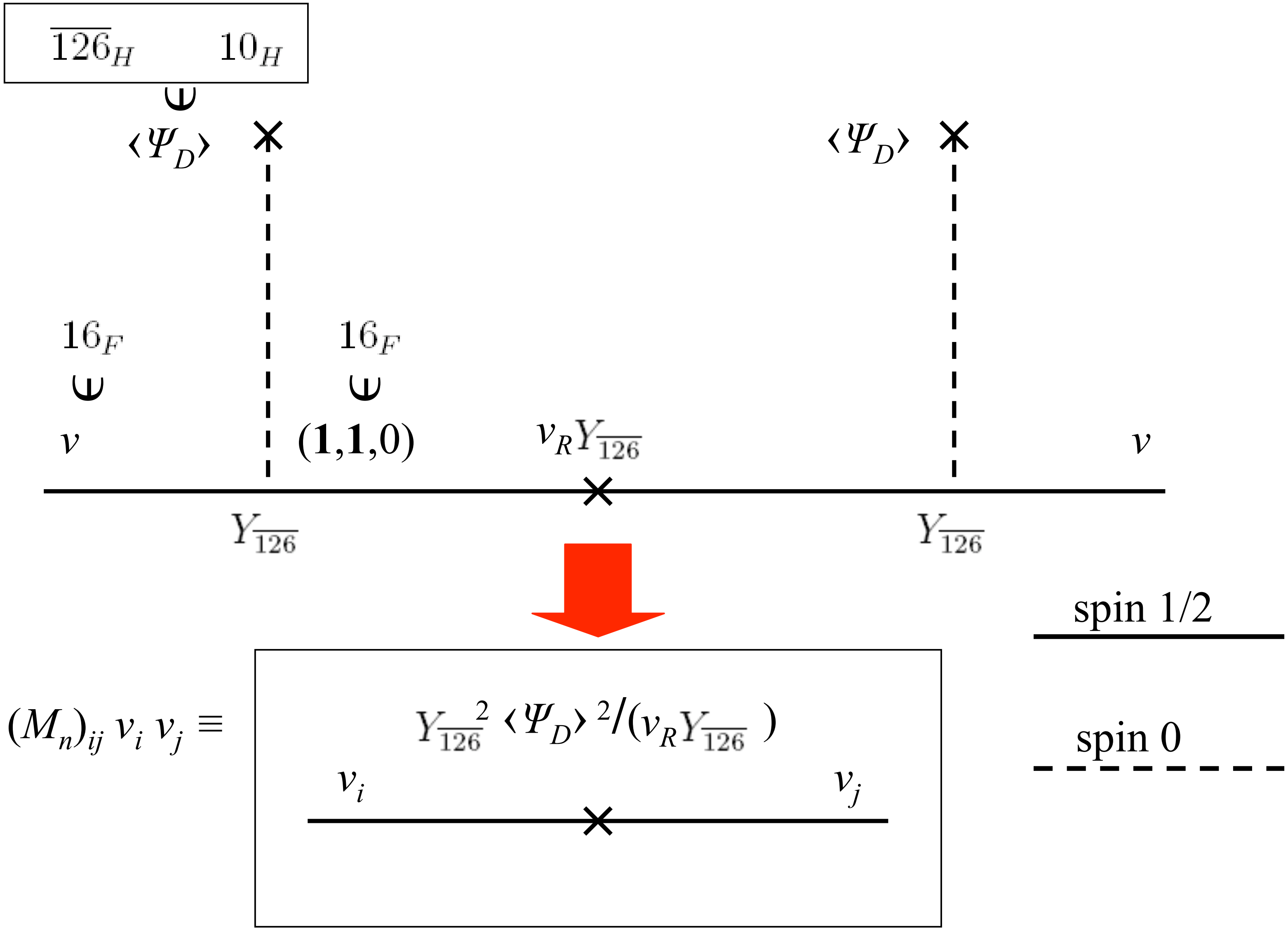
"Type I seesaw": $m_I = M_e(M_d - M_e)^{-1}M_e - 6\xi M_e + 9\xi^2(M_d - M_e)$

"Type II seesaw": $m_{II} = M_d - M_e$

OUR CONVENTION: $\hat{M}_j = U_j^T M_j U_j,\ j = u, d, e, n$

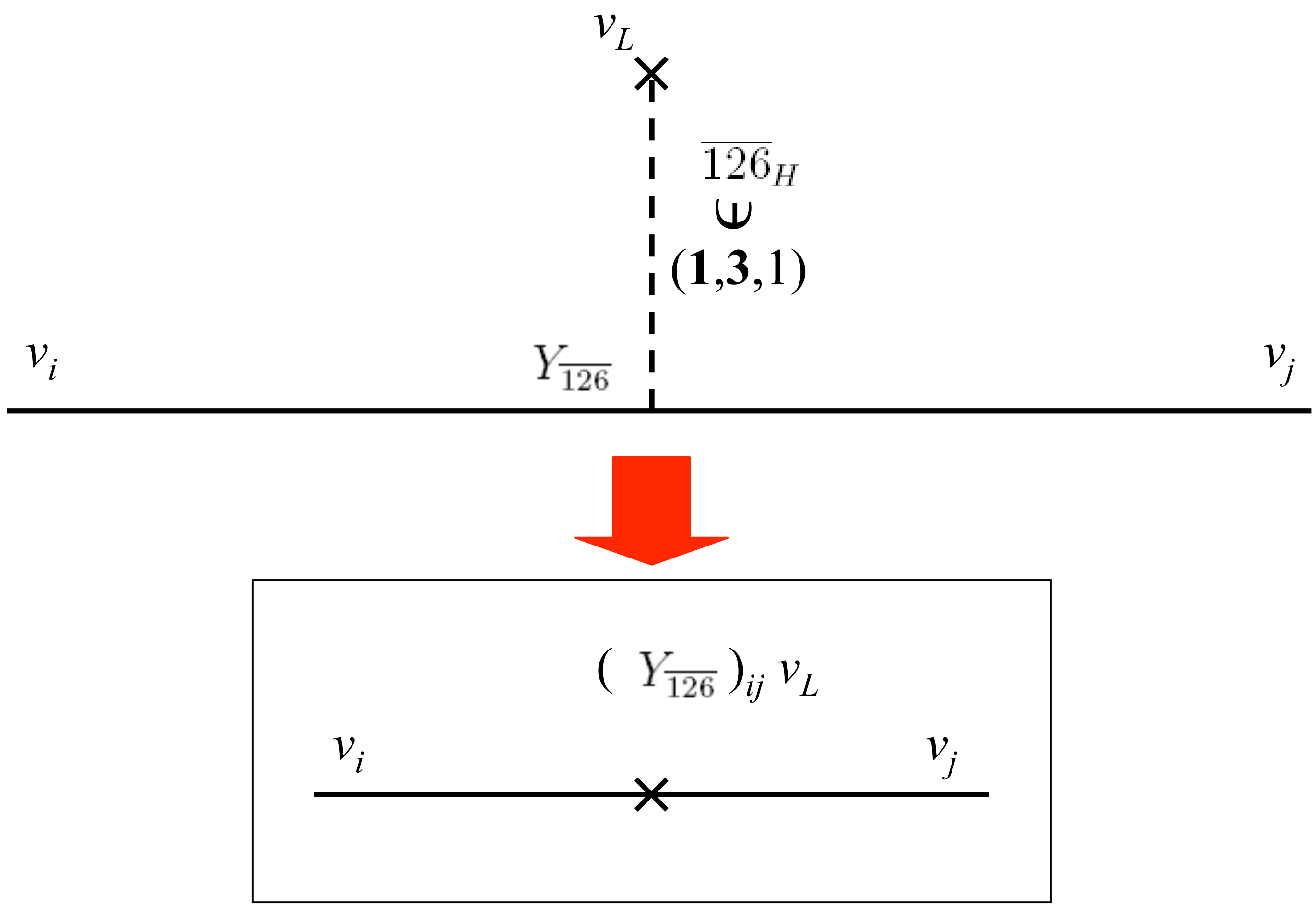
*B. Bajc, A. Melfo, G. Senjanović and F. Vissani, hep-ph/0511352.

NEUTRINO MASS — "TYPE I SEESAW"



$v_R \equiv$ vacuum expectation value

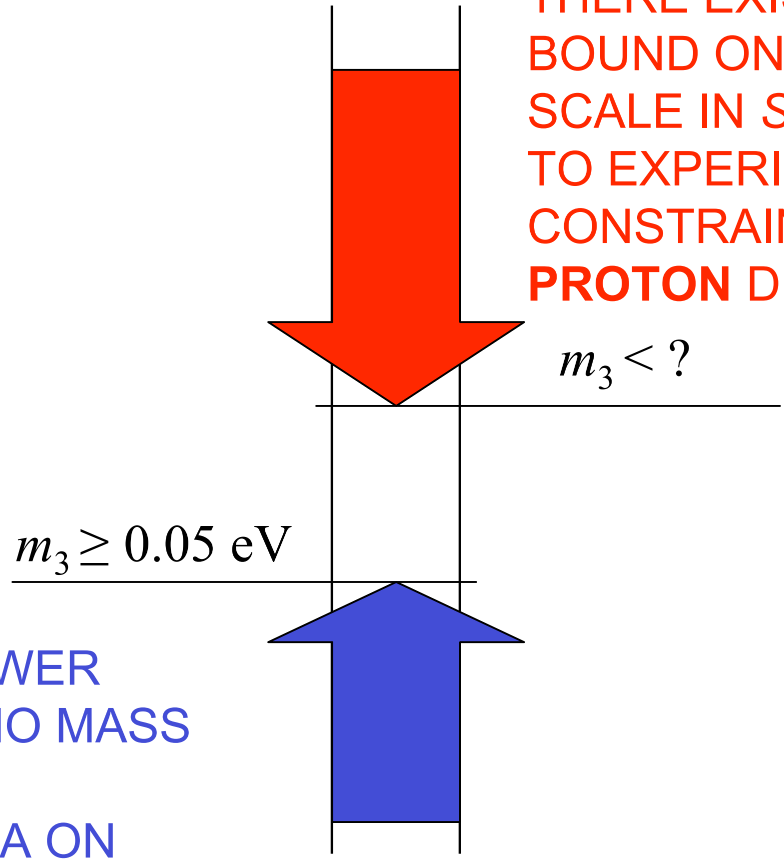
NEUTRINO MASS — "TYPE II SEESAW"



$v_L \equiv$ vacuum expectation value

NEUTRINO MASS SCALE PROBLEM IN MINIMAL *SO*(10)

THERE EXISTS AN UPPER BOUND ON NEUTRINO MASS SCALE IN *SO*(10) MODEL DUE TO EXPERIMENTAL CONSTRAINTS ON THE $d = 6$ **PROTON** DECAY OPERATORS!



THERE EXISTS A LOWER BOUND ON NEUTRINO MASS SCALE DUE TO EXPERIMENTAL DATA ON **NEUTRINO** OSCILLATIONS!

"TYPE II SEESAW" NEUTRINO MASSES

$$M_n = F_I\, m_I + \underline{F_{II}\, m_{II}}$$

$$m_{II} = M_d - M_e$$

$$\boxed{F_{II} = \frac{v}{m}\frac{\sin^2\beta}{\cos\beta}\alpha\sqrt{\frac{|\lambda|}{|\eta|}}\frac{N_u^2}{N_d}|f_{II}|}$$

*B. Bajc, A. Melfo, G. Senjanović and F. Vissani, hep-ph/0511352.

“TYPE II SEESAW” & NEUTRINO MASS

LOWER BOUND:

$$F_{II} \geq 0.2 \times 10^{-9}$$

b–τ **UNIFICATION**

UPPER BOUND:

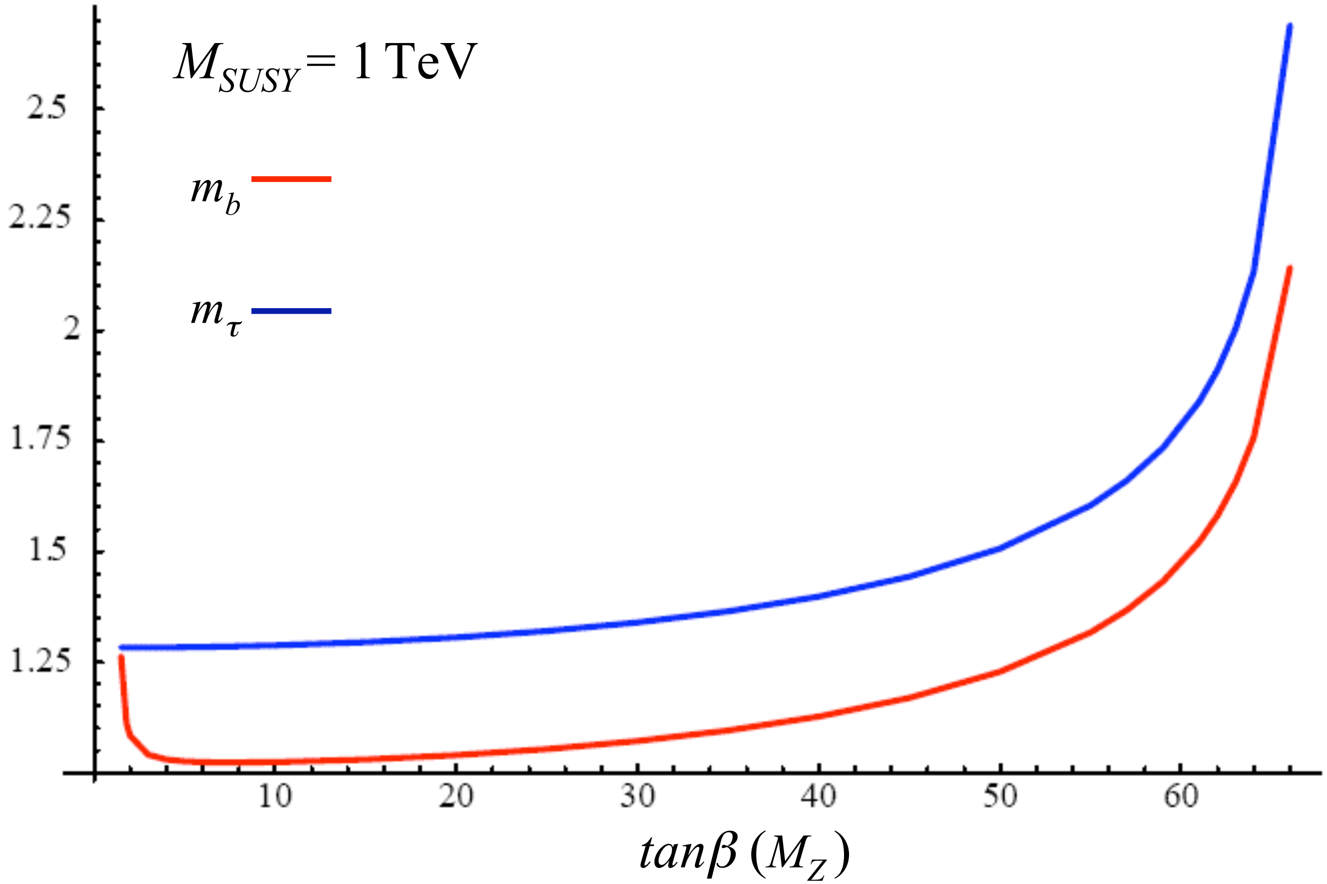
$$\frac{1}{m} < \frac{f(x)}{|\lambda|} \frac{1}{4 \times 10^{15}\,\mathrm{GeV}}$$

$$M_{(X,Y)} = m \frac{g_{GUT}}{|\lambda|} \sqrt{4 \left| \frac{1 - x - 2x^2}{x - 1} \right|^2 + 2 \left| \frac{2x(2x^2 + x - 1)}{(x-1)^2} \right|^2} = m g_{GUT} \frac{f(x)}{|\lambda|}$$

PROTON DECAY ($d = 6$)

b–τ UNIFICATION

b and τ masses at M_{GUT} in GeV units



$|m_b\text{-}m_\tau| \approx 0.25 \times 10^9$ eV

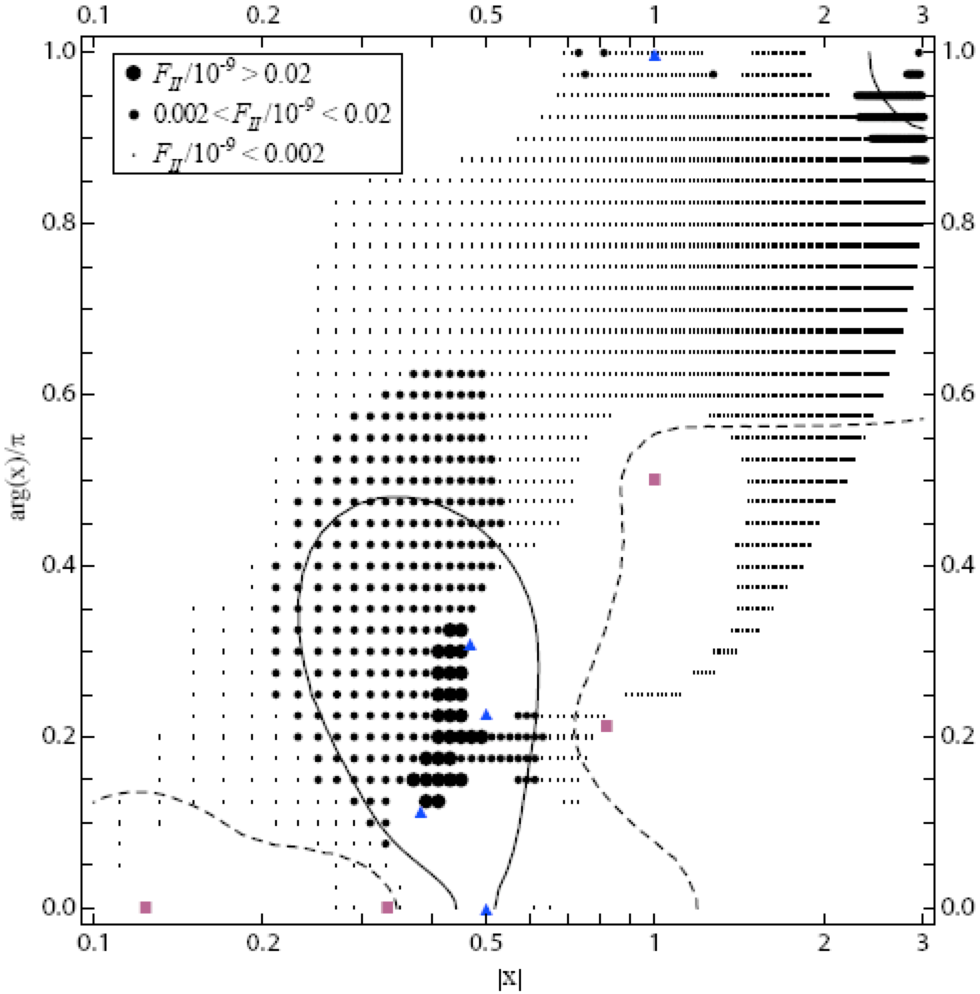
$F_{II} \geq 0.2 \times 10^{-9}$

$F_{II} \times (m_b\text{-}m_\tau)$

$m_3 \geq 0.05$ eV

LOW SUSY SCALE & ν MASS

$$\eta = \lambda = \alpha = \overline{\alpha} = 1 \qquad \tan\beta = 50$$



$\blacktriangle \equiv f_{II}(x)$ singularities

$\blacksquare \equiv f_{I}(x)$ singularities

——— $\equiv F_{II}^{\max}/10^{-9}=0.02$

- - - - - $\equiv F_{II}^{\max}/10^{-9}=0.002$

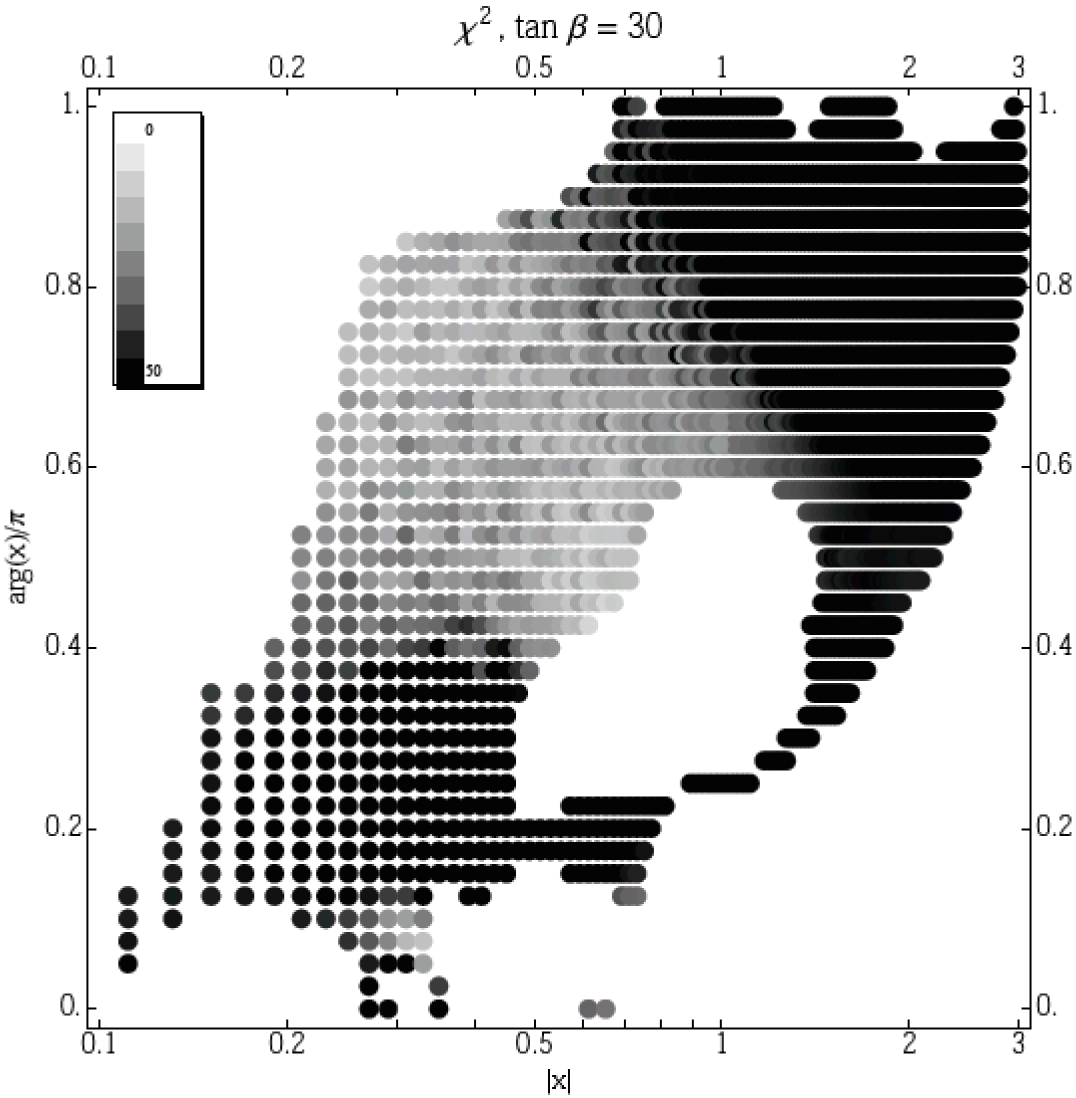
NUMERICAL PROCEDURE

$$\chi_i = \frac{p_i - \tilde{p}_i}{f_i \tilde{p}_i} \quad \Rightarrow \quad \chi^2 = \chi_q^2 + \chi_\ell^2, \quad \chi_q^2 = \sum_{i=1}^{10} \left(\frac{p_i - \tilde{p}_i}{f_i \tilde{p}_i} \right)^2$$

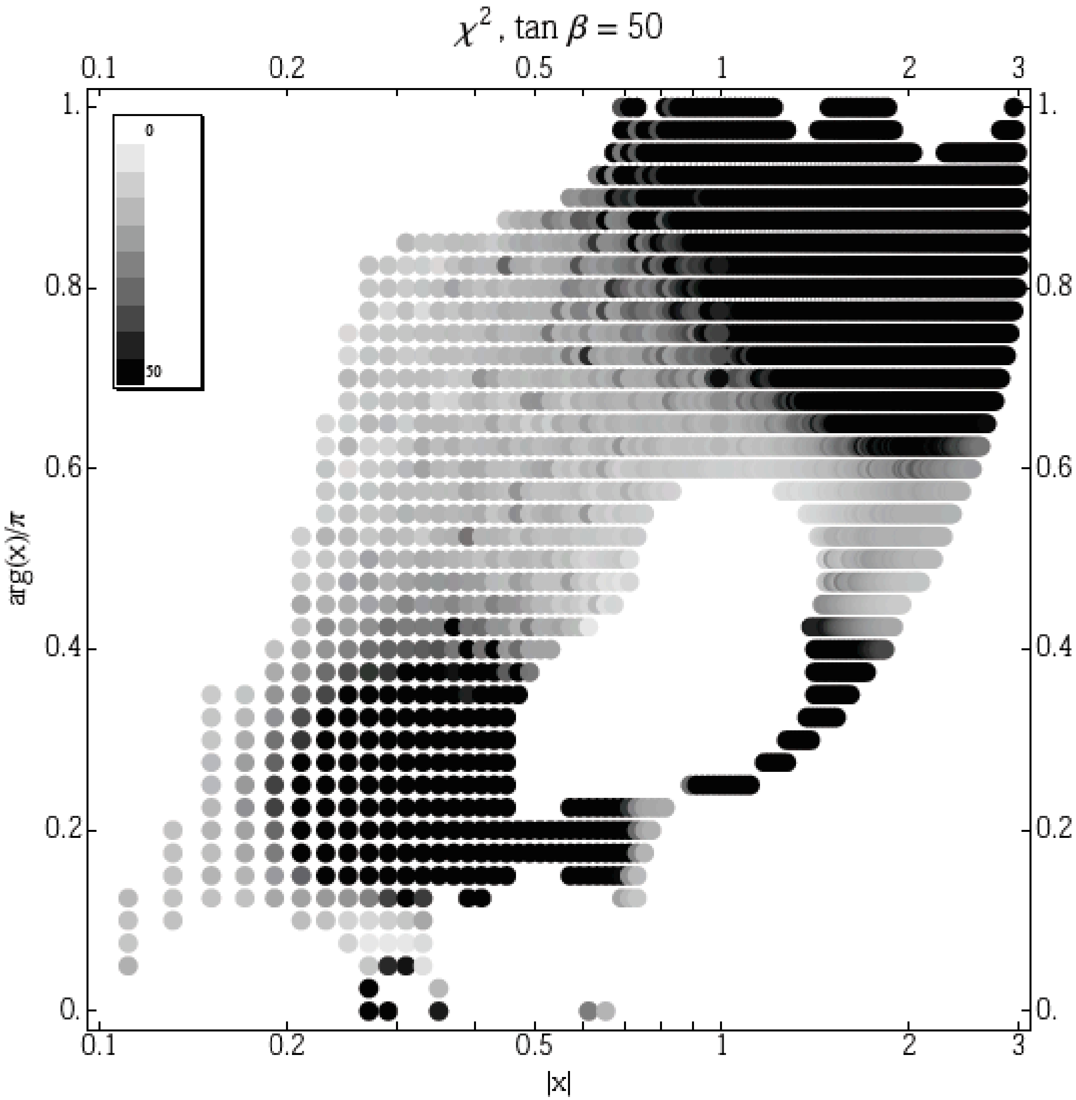
i	1	2	3	4	5	6	7	8	9	10
p_i	m_u	m_c	m_t	m_d	m_s	m_b	s_{12}^{CKM}	s_{23}^{CKM}	s_{13}^{CKM}	δ^{CKM}
f_i (%)	30	10	10	35	35	10	7	4	1	20

$$\chi_\ell^2 = \left(\frac{m_2/m_3 - 0.18}{0.02} \right)^2 + \left(\frac{s_{12}^{PMNS} - 0.5}{0.15 \times 0.5} \right)^2 + \left(\frac{s_{23}^{PMNS} - 0.6}{0.1 \times 0.6} \right)^2 + \left(\frac{s_{13}^{PMNS} - 0.17}{0.2 \times 0.17} \right)^2 \Theta\left(s_{13}^{PMNS} - 0.17\right)$$

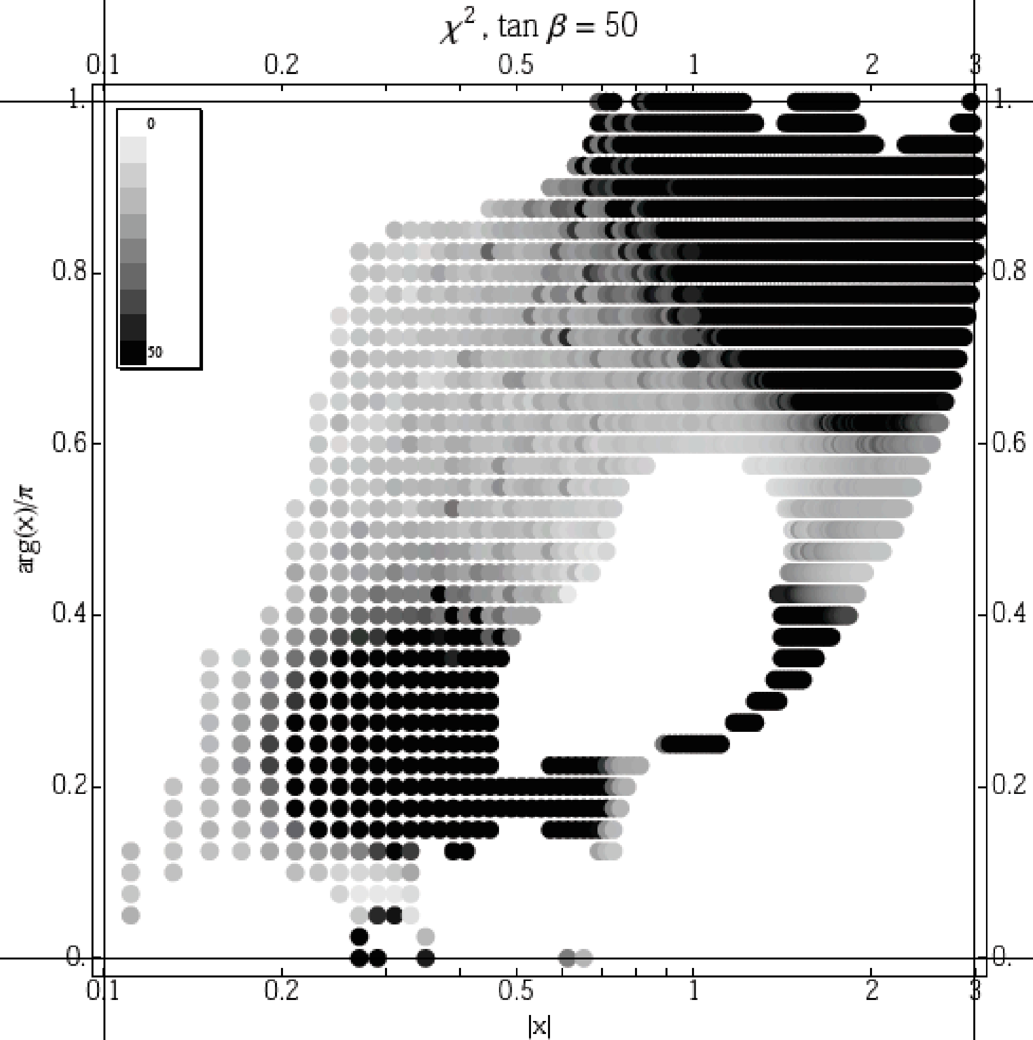
Notation: $s_{12}^{CKM} = \sin(\theta_{12}^{CKM})$ …



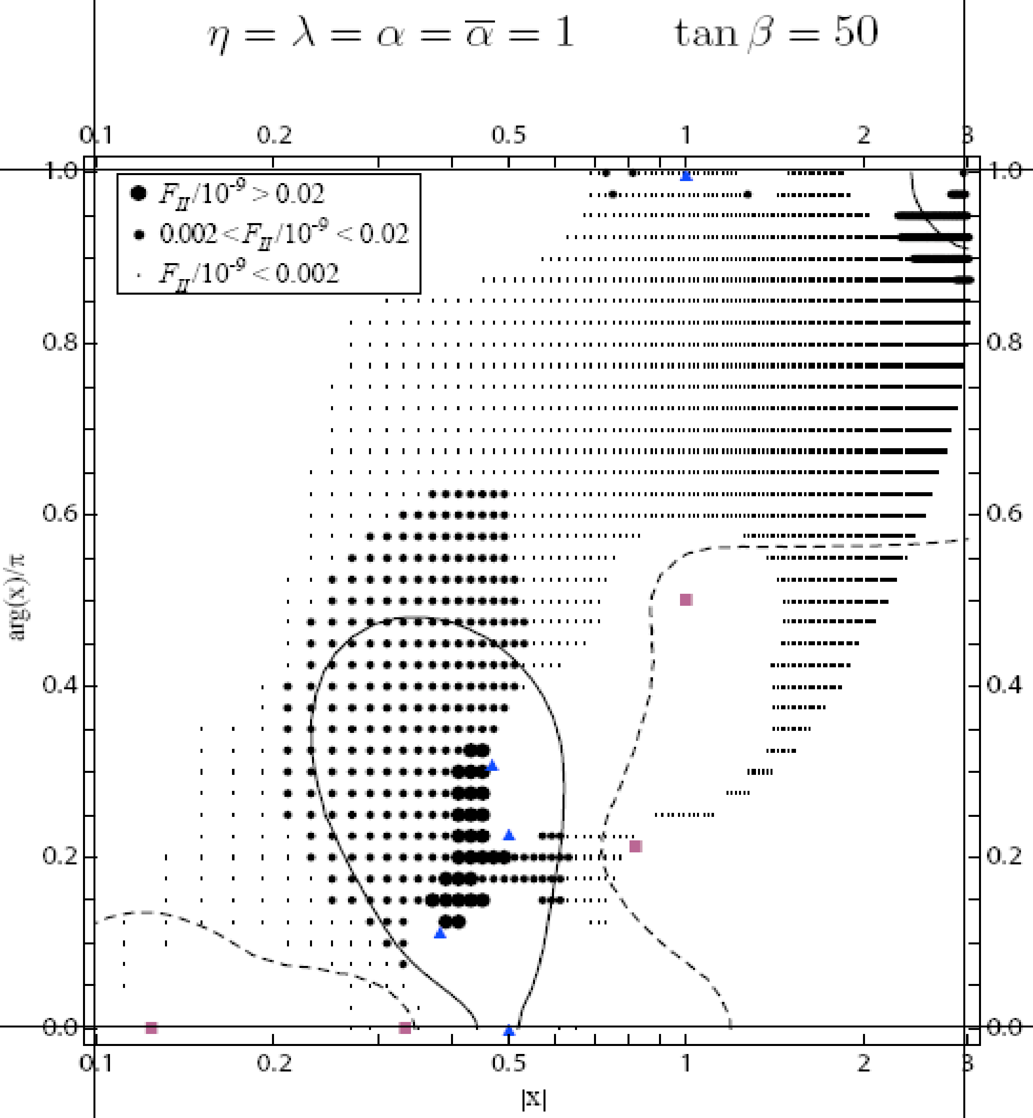
χ^2, tan $\beta = 30$
0.1
0.2
0.5
1
2
3
1.
0.8
0.6
0.4
0.2
0.
0
50
arg(x)/π
|x|



χ^2, tan $\beta = 50$
0.1
0.2
0.5
1
2
3
1.
0.8
0.6
0.4
0.2
0.
0
50
arg(x)/π
|x|



χ^2, tan $\beta = 50$
0.1
0.2
0.5
1
2
3
1.
0.8
0.6
0.4
0.2
0.
0
50
arg(x)/π
|x|



$\eta = \lambda = \alpha = \overline{\alpha} = 1 \qquad \tan\beta = 50$
$F_{II}/10^{-9} > 0.02$
$0.002 < F_{II}/10^{-9} < 0.02$
$F_{II}/10^{-9} < 0.002$
arg(x)/π
|x|
0.1
0.2
0.5
1
2
3
0.0
0.2
0.4
0.6
0.8
1.0

IMPORTANT FINDINGS...

$m: \quad 4 \times 10^{14}\,\mathrm{GeV} \;\; — \;\; 4 \times 10^{17}\,\mathrm{GeV}$

$\tan\beta = 30$

	m_u	m_c	m_t	m_d	m_s	m_b	m_e	m_μ	m_τ
min	.00053	.210	86.7	.0012	.0213	1.17	.000341	.0720	1.28
max	.00060	.237	93.2	.0014	.0240	1.29	.000367	.0775	1.37
MSSM	.00050	.198	75.7	.0011	.0202	1.07	.000357	.0754	1.34

$(0.05\,\mathrm{eV}/m_3) \geq 57 \qquad \chi^2/\mathrm{d.o.f.} < 10/13$

NOTE: Charged fermion masses are given in GeV units.

LOW SUSY SCALE IN MINIMAL *SO*(10) – STATUS

According to our findings minimal *SO*(10) model with low SUSY scale cannot offer satisfactory description of known fermion masses.

“SPLIT SUSY”* & NEUTRINO MASS

RELEVANT STEPS:

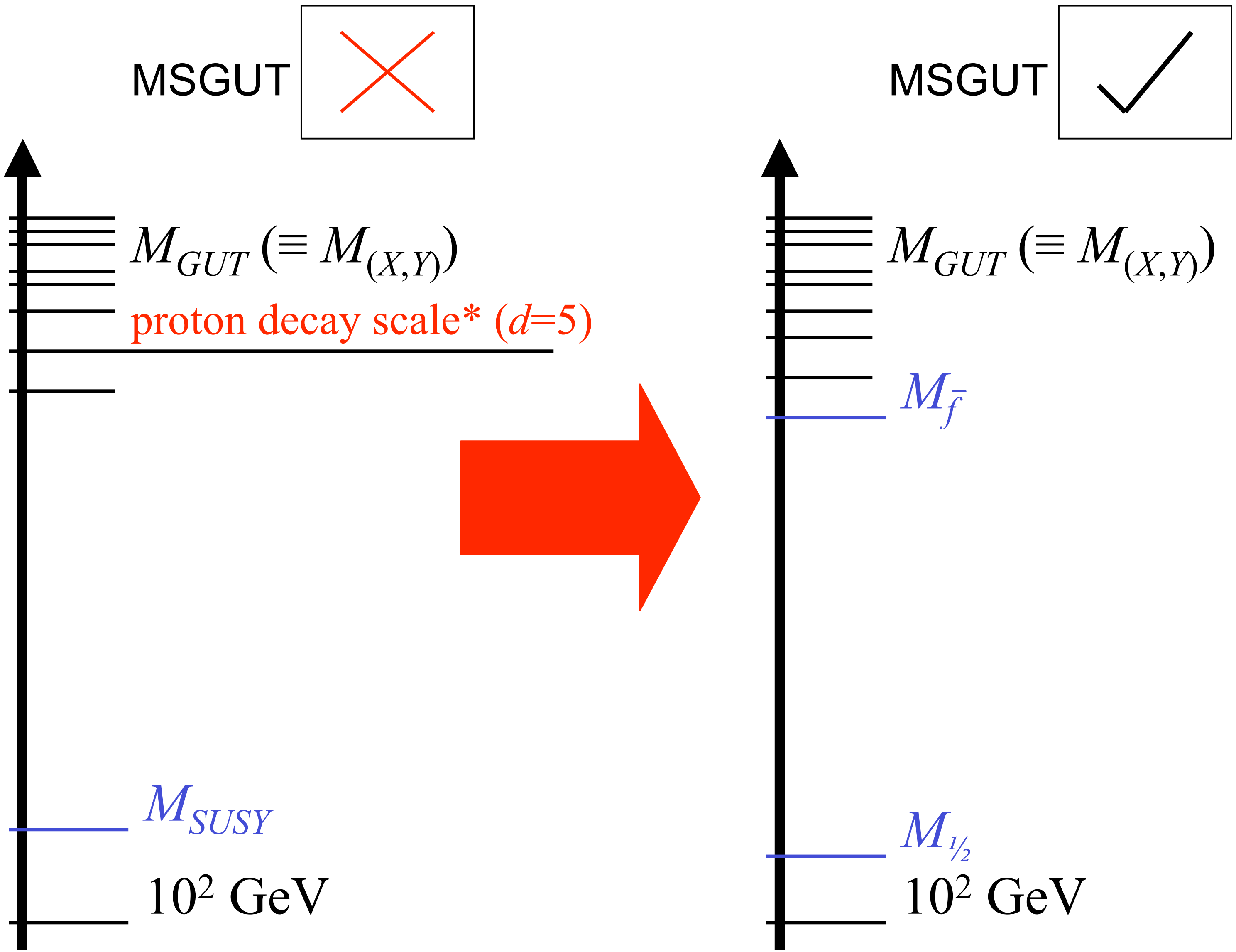
i) Find large enough F_{II} to describe neutrino mass scale. (This step completely determines x, m (M_{GUT}), α, $\overline{\alpha}$, λ i η.)

ii) Check whether the gauge coupling constants unify by solving the relevant renormalization group equations for three unknowns — g_{GUT}, $M_{\tilde{f}}$ and $M_{½}$.

iii) Propagate values of quark and charged lepton masses, and CKM parameters to M_{GUT} for given *tanβ* and minimize χ^2, which also includes neutrino masses (m_2/m_3) and the so-called solar and atmospheric angles.

*N. Arkani-Hamed and S. Dimopoulos, hep-ph/0405159; G. F. Giudice and A. Romanino, hep-ph/0406088.

“low SUSY” case vs. “split SUSY” case



*T. Fukuyama et al, hep-ph/0406068; B. Dutta et al, 0712.1206 [hep-ph].

NUMERICAL RESULTS

observable	data at M_Z	FIT	RGE	pull
m_e(MeV)	0.4866613	0.42279	0.42279	
m_μ(GeV)	0.10273	0.08925	0.08925	
m_τ(GeV)	1.746	1.534	1.534	
m_u(MeV)	1.6	0.56	0.54	+0.069
m_c(GeV)	0.628	0.218	0.214	+0.182
m_t(GeV)	171.5	72.0	67.4	+0.675
m_d(MeV)	3.5	0.24	1.2	−2.310
m_s(MeV)	62	27.2	21.8	+0.696
m_b(GeV)	2.89	0.917	0.910	+0.079
s_{12}^{CKM}	0.2272	0.2423	0.2272	+0.951
s_{23}^{CKM}	0.0422	0.0478	0.0474	+0.199
s_{13}^{CKM}	0.00399	0.00447	0.00448	−0.015
δ^{CKM}	0.995	1.149	0.995	+0.598
s_{12}^{PMNS}		0.42	0.55	−1.091
s_{23}^{PMNS}		0.55	0.69	−0.764
s_{13}^{PMNS}		0.103		
m_2/m_3		0.178	0.180	−0.104

NUMERICAL RESULTS

Table 2: Input (RGE) and output (FIT) parameters of the numerical fit at the GUT scale with $\chi^2/\text{d.o.f.} = 9.6/13$, $m_3 = 0.049\,\text{eV}$, $m_2 = 0.0087\,\text{eV}$ and $m_1 = 0.0012\,\text{eV}$. Unification takes place for $|x| = 0.109$, $arg(x)/\pi = 0.52$, $\alpha = 1.26788$, $\overline{\alpha} = 7$, $\eta = 6.54112$, $\lambda = 0.03$, $M_{1/2} = 1.5 \times 10^5$ GeV, $M_{\tilde{f}} = 9.0 \times 10^{13}$ GeV, $M_{GUT} = 5.8 \times 10^{15}$ GeV and $g_{GUT} = 1.3$ ($m = 6.5 \times 10^{13}$ GeV). The input data at M_Z are also given in the second column.

PROTON DECAY ($d = 6$)

THEORY: ($p \longrightarrow \pi^0 e^+$):

$$\Gamma = \frac{m_p}{16\pi f_\pi^2} A_L^2 |\alpha|^2 (1+D+F)^2 \left[A_{S\,R}^2 \left| k_1^2 (W_e^\dagger)^{11} + k_2^2 (V_q^\dagger)^{11} (V_q W_e^\dagger)^{11} \right|^2 + A_{S\,L}^2 k_1^4 \left| (W_e)^{11} + (V_q)^{11} (W_e V_q^\dagger)^{11} \right|^2 \right],$$

$$W_e = \begin{pmatrix} -0.9989 + 0.0260\,i & -0.0288 + 0.0227\,i & +0.0136 - 0.0023\,i \\ -0.0132 - 0.0348\,i & +0.8338 + 0.5478\,i & -0.0559 - 0.0156\,i \\ -0.0096 - 0.0080\,i & -0.0323 - 0.0486\,i & -0.7749 - 0.6293\,i \end{pmatrix}$$

$$V_q = \begin{pmatrix} +0.8936 + 0.3778\,i & -0.2298 - 0.0771\,i & -0.0011 + 0.0043\,i \\ -0.2386 - 0.0412\,i & -0.9648 - 0.0909\,i & -0.0438 + 0.0191\,i \\ +0.0106 + 0.0002\,i & +0.0440 + 0.0159\,i & -0.9828 + 0.1785\,i \end{pmatrix}$$

$$\mathrm{A_{SL}} = 2.0, \qquad \mathrm{A_{SR}} = 2.2$$

PROTON DECAY*

	Partial mean life (10^{33} years)		
p decay modes	MODEL	Lifetime bounds:	Fraction (Γ_i/Γ)
$p \to \pi^0 e^+$	1.6	> 1.6	44.3 %
$p \to \pi^0 \mu^+$	70	$> .473$	1.0 %
$p \to K^0 e^+$	442	$> .150$	0.2 %
$p \to K^0 \mu^+$	15	$> .120$	4.7 %
$p \to \eta e^+$	238	$> .313$	0.3 %
$p \to \eta \mu^+$	238	$> .313$	0.5 %
$p \to \pi^0 \bar{\nu}$	1.5	$> .025$	49.0 %

All relevant phases are determined through numerical procedure!
For example, $\delta^{PMNS} = 6.2$!
All relevant masses, for example, $M_{(X,Y)}$, are fixed through unification of gauge couplings α_i!

*P. Fileviez Pérez, hep-ph/0403286.

CONCLUSIONS

- **According to our findings minimal $SO(10)$ model with low SUSY scale cannot describe known fermion masses in satisfactory fashion.**

- **"Split SUSY" scenario not only provides correct neutrino mass scale but it also describes all masses and mixing parameters correctly.**

- **Model predicts *i*) proton decay lifetime that is well within the reach of the next generation of experiments and *ii*) experimentally accessible value for s_{13}^{PMNS}.**

CONCLUSIONS

Minimal *$SO(10)$* is still viable (and predictive) candidate of Grand Unification